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Encryption of Dual Numbers Using Edge Injective Labeling

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ABSTRACT

In this Research paper we examine the encryption technique by defining new functions on the face wheel graph. Instead of passing a message we have used digits as a message. These digits will be sealed by function of labeling. The resulting graph is represented as a Cipher Graph which is transferred to the receiver. Decryption process is done using a matrix algorithm by defining the matrix $K_{p\times\eta}$, $L_{p\times\eta}$ *and* $Z_{p\times\eta}$. *Graph Labeling is recognized as the truly unbreakable and provides absolute security of digits transmission.*

KEYWORDS: *Vertex Labeling,Edge Injective Labeling, Face Wheel graph, Matrix inversion.*

2010 Mathematical subject classification Number: 05C78.

INTRODUCTION

Graph Theory plays an important role in various fields like coding theory, Networking, Cryptography and chemical coding. One of the important fields of graph theory is labeling. Graph Labeling is the number allotted to the vertex and edges of a graph. Labeling is divided into two parts of function: vertex labeling and edge labeling. Mostly we define the label as integers. Vertex labeling is assigning the number to the vertex in a graph and edge labeling is assigning the number to the edge in the graph. In the year 1967 Alexander Rosa developed the concept of graph labeling. Alexander Rosa defined the three labeling α, β and γ. Later β labeling is named as Graceful labeling by Solomom Golomb. This labeling gave a new perspective to the rise of many different labeling. Now we have edge graceful labeling, Harmonious labeling, lucky labeling, Edge magic labeling, vertex magic labeling and so on.

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The word crypto system is given to the coding by the official U.S national security agency. The message elements are word, digits, Phrases or sentences. The converted ciphertext is called code groups. Nowadays graph labeling is used in secure communication of messages. Confidentiality of message transferring becomes very important and also challenging. This research work involves the digits converted into a cipher graph and matrix.

RELATED WORKS

The method of graph labeling was developed by Rosa in 1967. Later researchers like J.Baskar Babujee [4] [5] [6]gave an approach to Edge Bimagic Total Labeling. He also extended his research on generalization of Edge Magic Total Labeling. Mohammed Ali and Babujee [11] developed the theory of Strong Face Graph and also developed the concept of bimagic on them. J.Baskar Babujee and S.Babitha[7] layed a ground works of Encrypting and decrypting number through labeled graphs. Getting motivated by the above research work we had encrypted the sealed numbers by FWG and decrypted by matrix.

2. MAIN RESULTS

2.1 DEFINITION

Let G(V,E) be a plane graph. A face graph G_F is defined by joining a current vertex to each face of G by leaving the outer face unconnected. Join all the vertices adjacent to the face of the G this leads to a new graph of cycle ζ_3

2.2 EDGES INJECTIVE LABEL (*EIL*)

Let G(V,E) be a graph with injection vertex set of V_F which results to induced edge set

$$\begin{Bmatrix} y = 3X - 1; & if\ 1 \leq y \leq 26 \\ y = (3X - 1) mod\ 26; & if\ y \geq 26 \end{Bmatrix}$$

where X is the sum of $u + v$ vertices.

2.3 ENCIPHER AND DECIPHER OF DUAL NUMBERS USING FWG

2.3.1 ALGORITHM FOR ENCIPHER

Message: The sender encode two positive sealed numbers R_1 and R_2, where R_1 have p digits and R_2 have q digits, $q \geq p$.

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Output: We get encrypted edge injective labeled FWG

1.Consider the vertex and edge set to be

$V_1(FW) = \{uv_j : j = 1,2,..n\}$, $V_2(FW) = \{u_j : j = 1,2,..n\}$

$V(FW) = V_1(FW) \cup V_2(FW)$

$$E(FW) = \begin{cases} 3X - 1; \\ (3X - 1) mod\ 26; \end{cases}$$ where X=sum of vertices

2. Let us define the following functions

$f(m) = 1$

$f(mOV_j) = j + 2$

$f(IV_j) = (q + 4) + j$

where $j = 1,2,..n$

Now we set the edges as $f(Ou_j)$, $f(I\zeta_1 u_j)$, $f(I\zeta_2 u_j)$

$f(Ou_j) = 3(j + 2) - 1$

If $f(Ou_j) > 26$ then calculate modulus of 26

$f(I\zeta_1 u_j) = 3\{(q + 4) + j) - 1$

$f(I\zeta_2 u_j) = 3\{(q + 4) + j) - 1$

Conditions

a. If $f(I\zeta_1 u_j) > 26$ then calculate modulus of 26

b. If $f(Ou_j) = f(I\zeta_1 u_j)$ then consider the value of $f(I\zeta_1 u_j)$ without modulus

c. If $f(I\zeta_2 u_j) > 26$ then calculate modulus of 26

d. If then take the value without modulus and interchange the numbers

e. After the step d still the $f(I\zeta_1 u_j) = f(I\zeta_2 u_j)$ or $f(I\zeta\ u_j) = f(Ou_j)$ then add 1 to the value

f. After the step e still the $f(I\zeta_1 u_j) = f(I\zeta_2 u_j)$ or $f(I\zeta\ u_j) = f(Ou_j)$ then subtract 1 to the value

3. Separate the first sealed number R_1 into p digits $q \geq p$, such that

a. If p=q then the each numbers of R_1 $e_j \geq 0$ where $1 \leq j \leq n$

b. If $p \geq q$ then numbers $e_{q+1}, e_{q+2}, \ldots e_n$ are empty spaces

4. Separate the second sealed number R_2 into q digits $R_2 = q_1 q_2 \ldots . q_n$ where $q_1 q_2 \ldots . q_n$ are the digits of R_2

5. Define $\phi : V(FW) \to N^{+}$ such that

$$\emptyset(V(FW)) = \left\{ \begin{matrix} f(mOV_j) = j+2 \\ f(IV_j) = (q+4)+j \end{matrix} \right\}$$

2.3.2 ALGORITHM FOR DECIPHER

Message: The receiver gets the FWG labeled graph picture

Output: The two sealed numbers $R_1$and R_2

1. Create a matrix $K_{p\times\eta}$, where $k_{ab} = \left\{ \begin{matrix} f(mOV_j) = j+2 \\ f(IV_j) = (q+4)+j \end{matrix} \right\}$ where $\eta = 2$

2. Construct a matrix $L_{p\times\eta}$, where $l_{ab} = \left\{ \begin{matrix} 2q-1 \quad for\ b = 1, a = 1,2 \ldots .n \\ q+1 \quad for\ b = 2, a = 1,2, \ldots .n \end{matrix} \right\}$

3. Create a matrix $T_{p\times\eta}$, where $t_{ab} = a+b$ for a+b where $b = 1,2$ and $a = 1,2..n$

4. Calculate the matrix $Z_{p\times\eta} = L_{p\times\eta} - T_{p\times\eta}$ where $b = 1,2$ and $a = 1,2..n$

After finding $Z_{p\times\eta}$ interchange the columns as $Z_{b\times a}$. Also screen the elements in $Z_{b\times a}$. If the elements in $Z_{b\times a}$ is

p=q then we change the elements as 2q+3 and 2q.

Now calculate $\psi_{p\times\eta} = K_{p\times\eta} - Z_{p\times\eta}$ for b=1,2 and a= 1,2,….n

5. $\psi_{p\times\eta}$ gives the two sealed numbers $R_1 = z_{11}, z_{21}, z_{31} \ldots z_{n1}$ and $R_2 = z_{n2}, \ldots . z_{32}, z_{22}, z_{12}$ respectively. Ignore the

negative numbers. Note that R2 is the reversed value of column 2

3. ILLUSTRATION

Encryption:

Let R_1=24681 and R_2=5408642 be two sealed numbers. The digits of R_1=p=5 and R_2=q=7. Let us consider the face wheel graph FW_6.

As per the encipher algorithm step 1 allocate the vertices labels to the graph FW.

$V_1(FW) = \{uv_j : j = 1,2,..n\}$, $V_2(FW) = \{u_j : j = 1,2,..n\}$

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$V(FW) = V_1(FW) \cup V_2(FW)$

$$E(FW) = \begin{cases} 3X - 1; \\ (3X-1) \bmod 26; \end{cases} \text{where } X = \text{sum of vertices}$$

Next, set edge values according to step 2 in the encipher algorithm. The encrypted FW_6 graph is transmitted to the receiver.

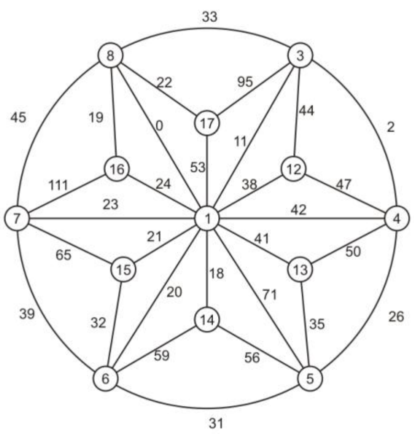
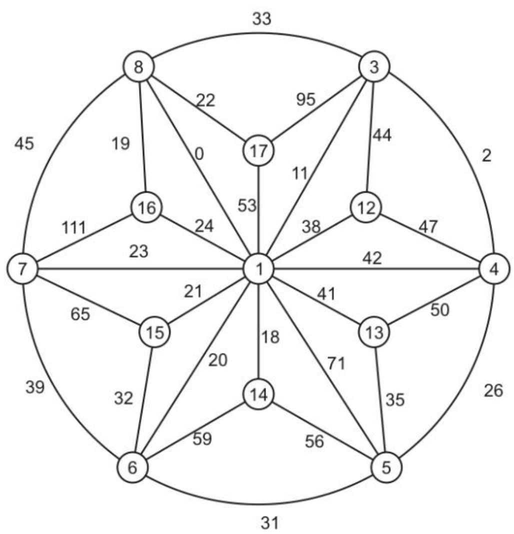


Figure 1- Cipher Graph

Decryption

The receiver gets the FW_6 labeled graph.



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As per the decipher algorithm step 1 we find the matrix $K_{6\times\eta} = \begin{bmatrix} 3 & 13 \\ 4 & 14 \\ 5 & 15 \\ 6 & 16 \\ 7 & 17 \\ 8 & 18 \\ 0 & 19 \end{bmatrix}$ where $\eta = 2$

From the step 2 and 3 we get the following matrices

$$L_{6\times\eta} = \begin{bmatrix} 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \end{bmatrix} T_{6\times\eta} = \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 9 \end{bmatrix}$$

As per the step 4 calculate the matrix $Z_{p\times\eta} = K_{p\times\eta} - \psi_{p\times\eta}$ where

$\psi_{p\times\eta} = K_{p\times\eta} - Z_{p\times\eta}$ for b=1,2 ; a= 1,2,….n ; $\eta = 2$

$$Z_{6\times\eta} = \begin{bmatrix} 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \\ 13 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 3 & 4 \\ 4 & 5 \\ 5 & 6 \\ 6 & 7 \\ 7 & 8 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} 11 & 5 \\ 10 & 4 \\ 9 & 3 \\ 8 & 2 \\ 7 & 1 \\ 6 & 0 \\ 5 & -1 \end{bmatrix}$$

$$\psi_{6\times\eta} = \begin{bmatrix} 3 & 13 \\ 4 & 14 \\ 5 & 15 \\ 6 & 16 \\ 7 & 17 \\ 8 & 18 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 5 & 11 \\ 4 & 10 \\ 3 & 9 \\ 2 & 8 \\ 1 & 17 \\ 0 & 14 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} -2 & 2 \\ 0 & 4 \\ 2 & 6 \\ 4 & 8 \\ 6 & 0 \\ 8 & 4 \\ 1 & 5 \end{bmatrix}$$

Step 5 we get the two sealed numbers $R_1 = z_{11}, z_{21}, z_{31} \ldots z_{n1} = [24681]$ and $R_2 = z_{12}, z_{22}, z_{32} \ldots z_{n2} = [5408642]$. Ignore the negative numbers.

CONCLUSION

We have developed a new encryption of dual numbers using FWG. This method will be useful to share a dual password in confidential to a graph. We can also create a new encryption method by varying the definition and algorithm.

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